

Une histoire d'énergie : équations et transition

Sustainable Energy, April 7th, 2026

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Joint work with Pr Damien Ernst - thanks to many other people

In the news

Quinze mille scientifiques alertent sur l'état de la planète

L'ampleur de l'initiative est sans précédent. Plus de 15 000 scientifiques de 184 pays signent un appel contre la dégradation catastrophique de l'environnement.

LE MONDE | 13.11.2017 à 16h05 • Mis à jour le 14.11.2017 à 14h18 |

Par Stéphane Foucart et Martine Valo

Airbus décroche la plus importante commande de l'histoire de l'aéronautique

L'avionneur a vendu au loueur américain Indigo 430 moyen-courriers A320 Neo pour une valeur de 42 milliards d'euros.

Le Monde.fr avec AFP | 15.11.2017 à 07h50 • Mis à jour le 15.11.2017 à 12h17 |

Par Guy Dutheil

La production de pétrole pourrait bientôt ne plus suffire

Par Armelle Bohineust | Mis à jour le 05/03/2018 à 15:54 / Publié le 05/03/2018 à 12:47

L'Agence internationale de l'énergie craint qu'après 2020 les capacités d'exploitation soient insuffisantes pour répondre à la hausse de la demande.

ÉCONOMIE • TRANSPORTS

Gigantesque commande de 300 avions Airbus par l'entreprise d'Etat chinoise CASC

Selon le « prix catalogue », le montant de cette commande de 290 Airbus A320 et de 10 A350 s'élève à près de 31,8 milliards d'euros.

Le Monde avec AFP et Reuters • Publié aujourd'hui à 18h38, mis à jour à 20h01

What does this mean to you?

Even more recently

« Ils évoquent la fin du monde, nous on parle de la fin du mois. Nous allons traiter les deux, et nous devons traiter les deux. »

E.M.

Présentation de la Programmation Pluriannuelle de l'Energie (PPE).

November 27th, 2018.

Outline

**Energy
Stories**

**Modeling
the
Transition**

Energy stories



**About 1 million years ago:
Fire domestication: lighting, heating, cooking
->improved health**



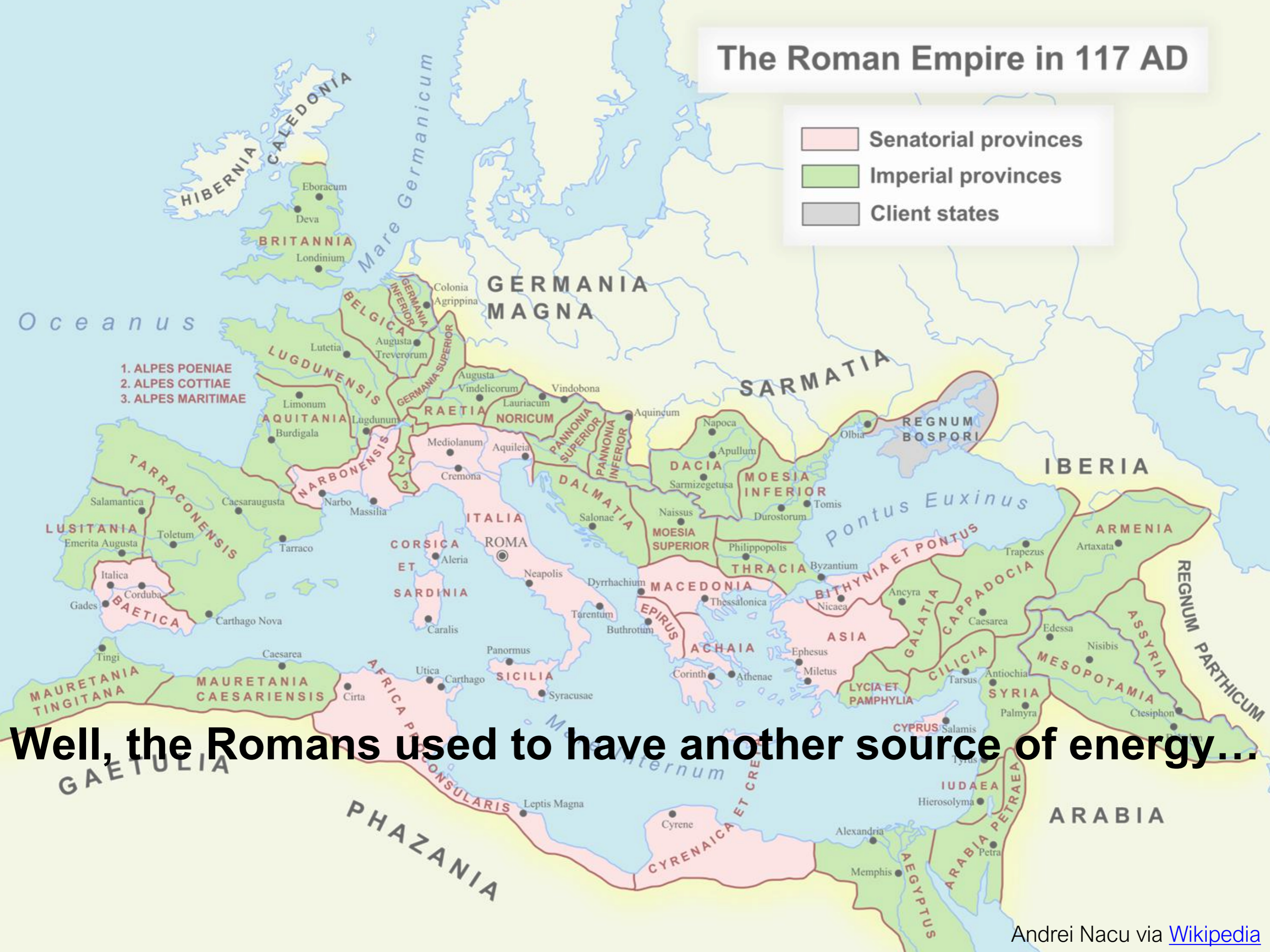
**About 10 000 years ago:
Agriculture: a 'new' way to 'efficiently' collect solar
energy via photosynthesis**

During the Roman Empire, agriculture provided food to humans (some of them are slaves) and animals: this was (almost) the only source of energy



The Roman Empire in 117 AD

- Senatorial provinces
- Imperial provinces
- Client states



Well, the Romans used to have another source of energy...

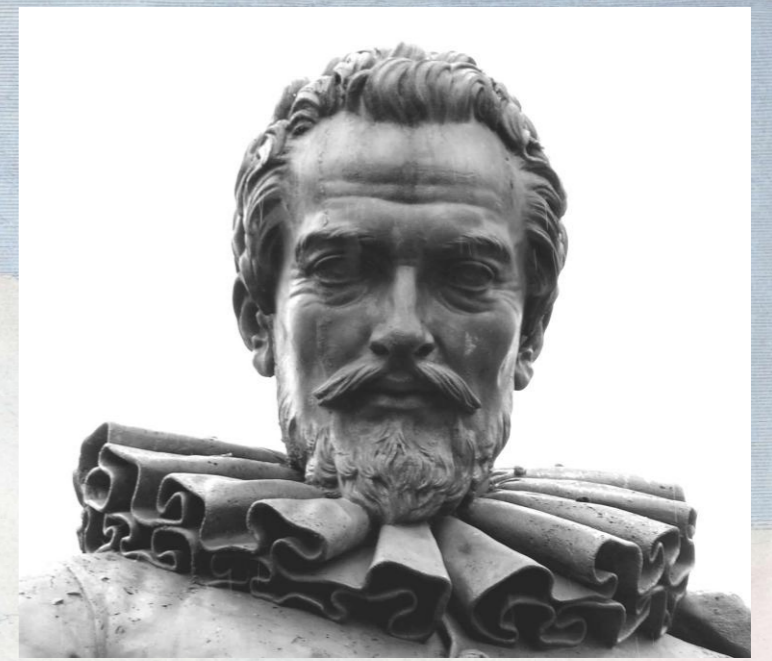


During the Middle Ages, mills are deployed in Europe
1 mill corresponds to (about) 40 men in terms of power
- European GDP*2 between 1000 and 1500
- « Only » 30% in Asia during the same period

A famous example: the Dutch Golden Age (16th century)

- Efficient agriculture
- Peat
- Waterways
- Trade, city development
- Sawmills for boat construction

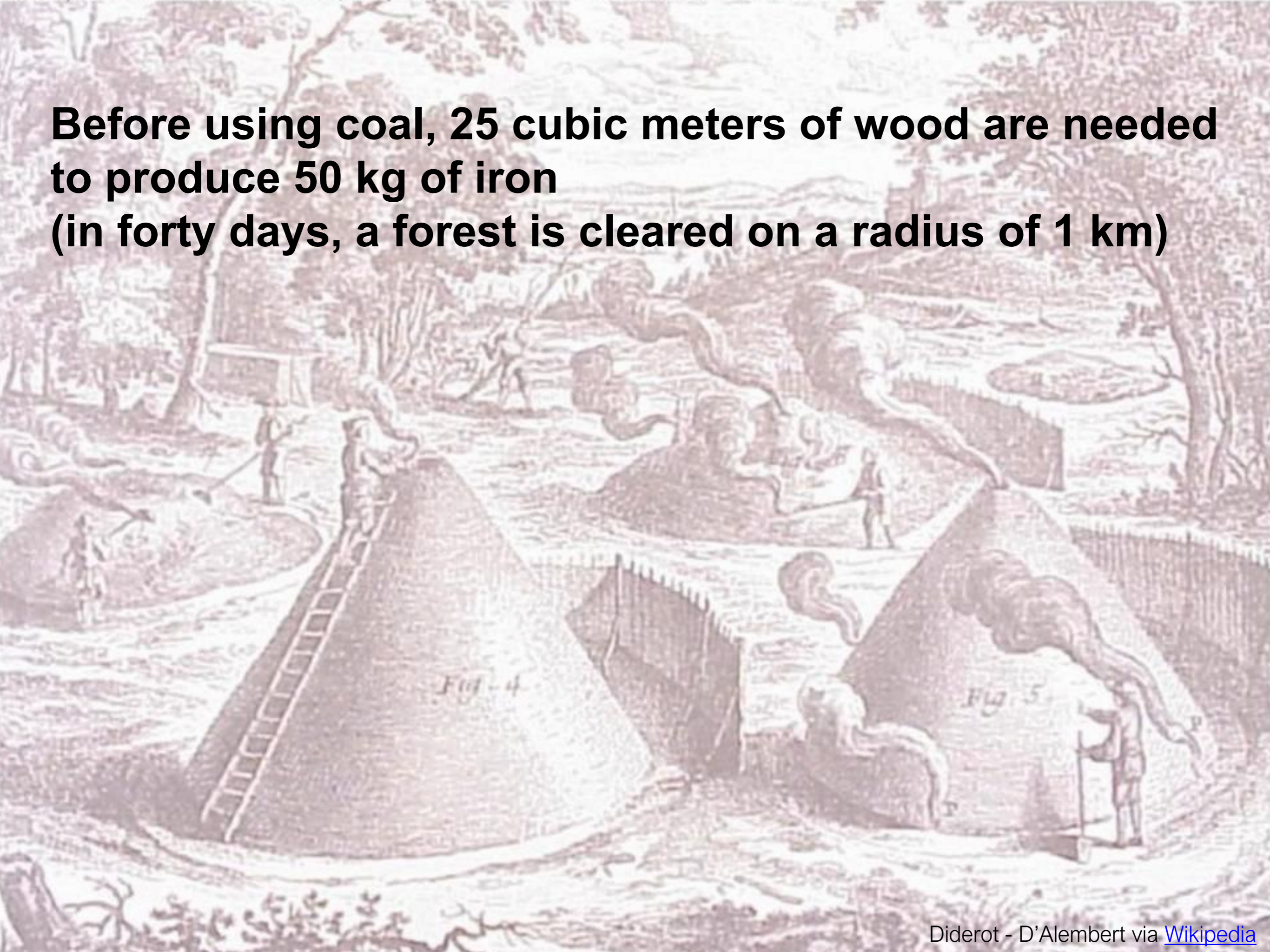




« Een Wonder en is gheen wonder »
Simon Stevin



**Before using coal, 25 cubic meters of wood are needed to produce 50 kg of iron
(in forty days, a forest is cleared on a radius of 1 km)**



In the UK, wood shortage leads to the discovery of the potential of coal

Coal made the massive development of metallurgy possible, leading to new infrastructures

After WW2, almost exponential growth of oil consumption opens the so-called « consumer society » era



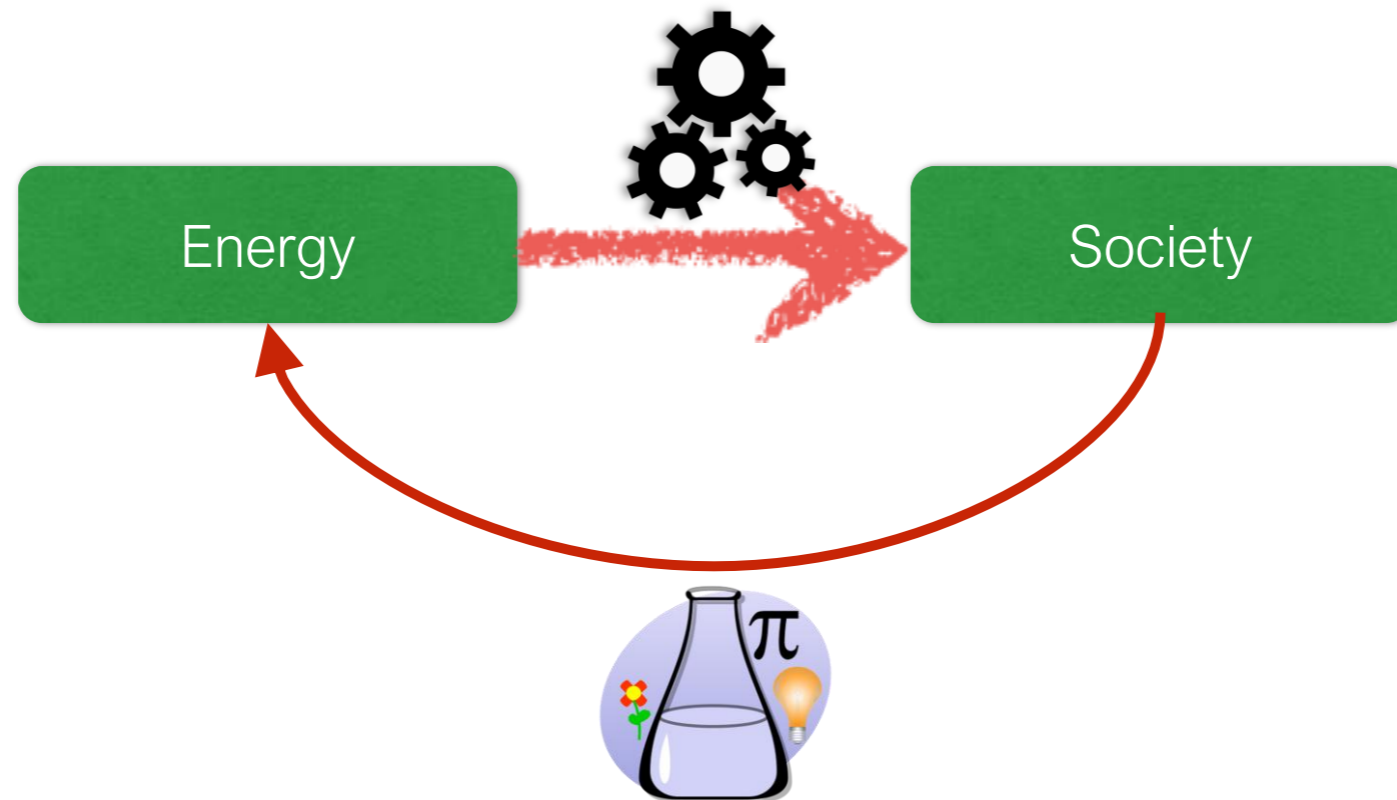
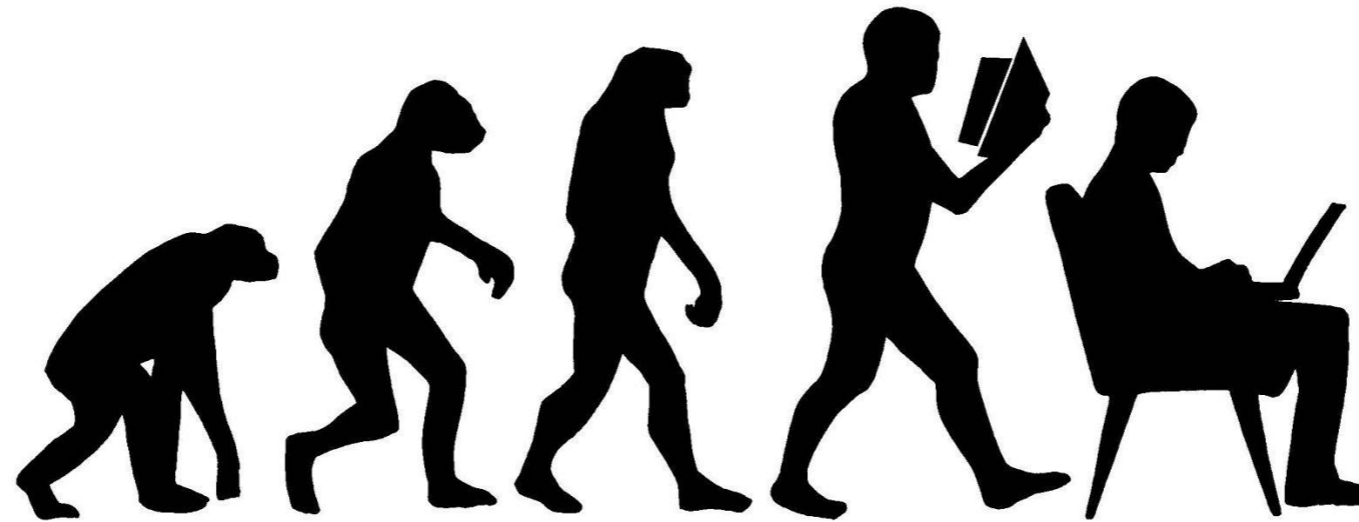
In Western Europe, almost 5% GDP growth per year during 30 years

« The Glorious Thirty » - « Les Trente Glorieuses »

-> 1973 Oil Crisis

-> In Europe, emergence of public debt and mass unemployment

Trajectories of Societies



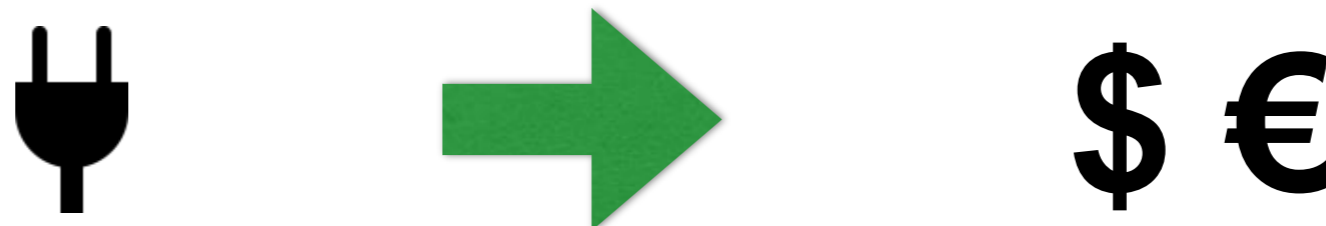
Energy & GDP

Recent research in Economics has shown that:

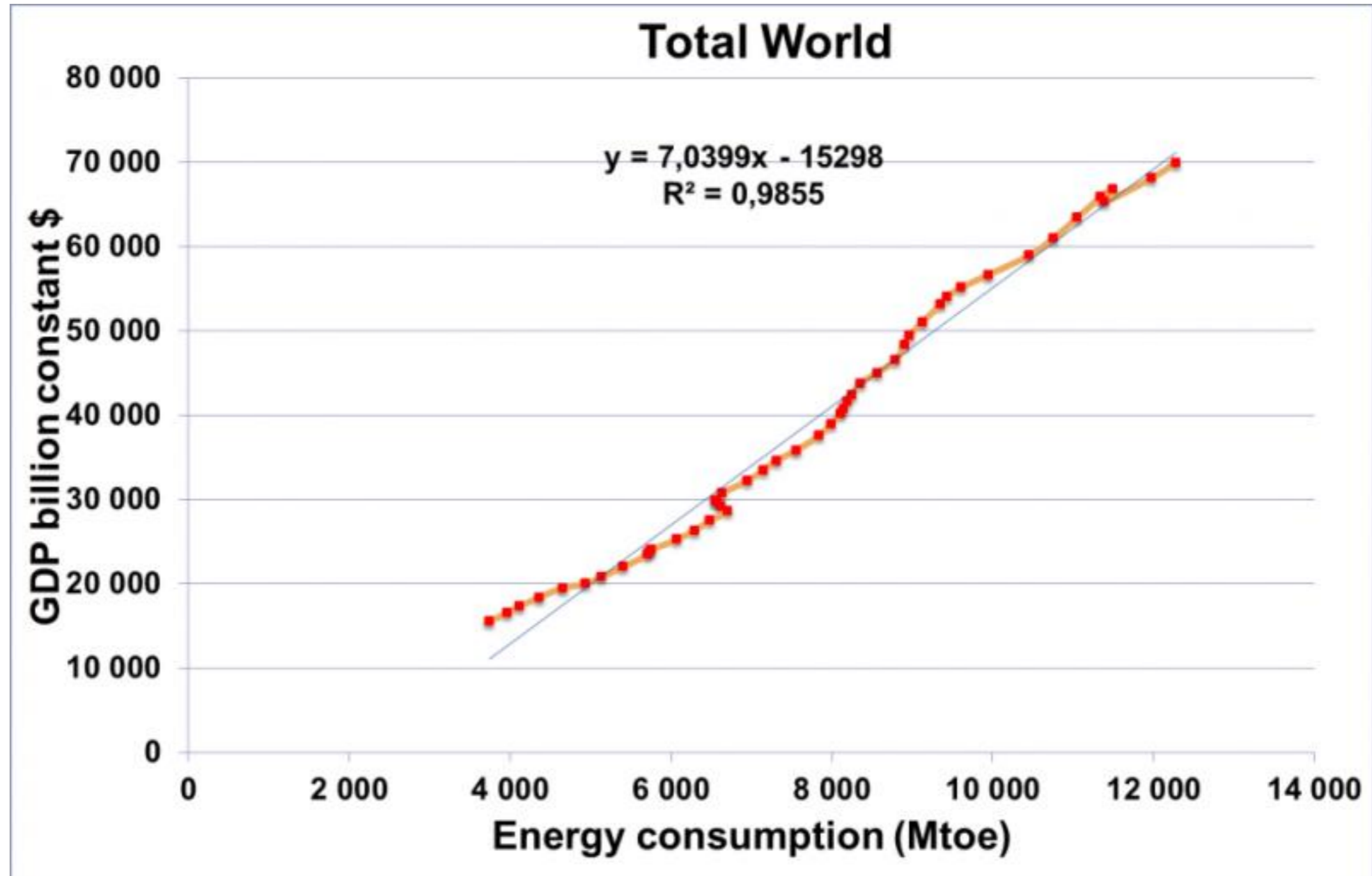
- The empirical elasticity (measured from time series among OECD countries over the last 50 years) of the consumption of primary energy into the GDP is about 60%, which is 10 times higher than what is predicted by the « *Cost Share Theorem* »

Elasticity can be quantified as the ratio of the percentage change in one variable to the percentage change in another variable

- There is a causality link between the consumption of primary energy and the GDP in the direction Energy -> GDP

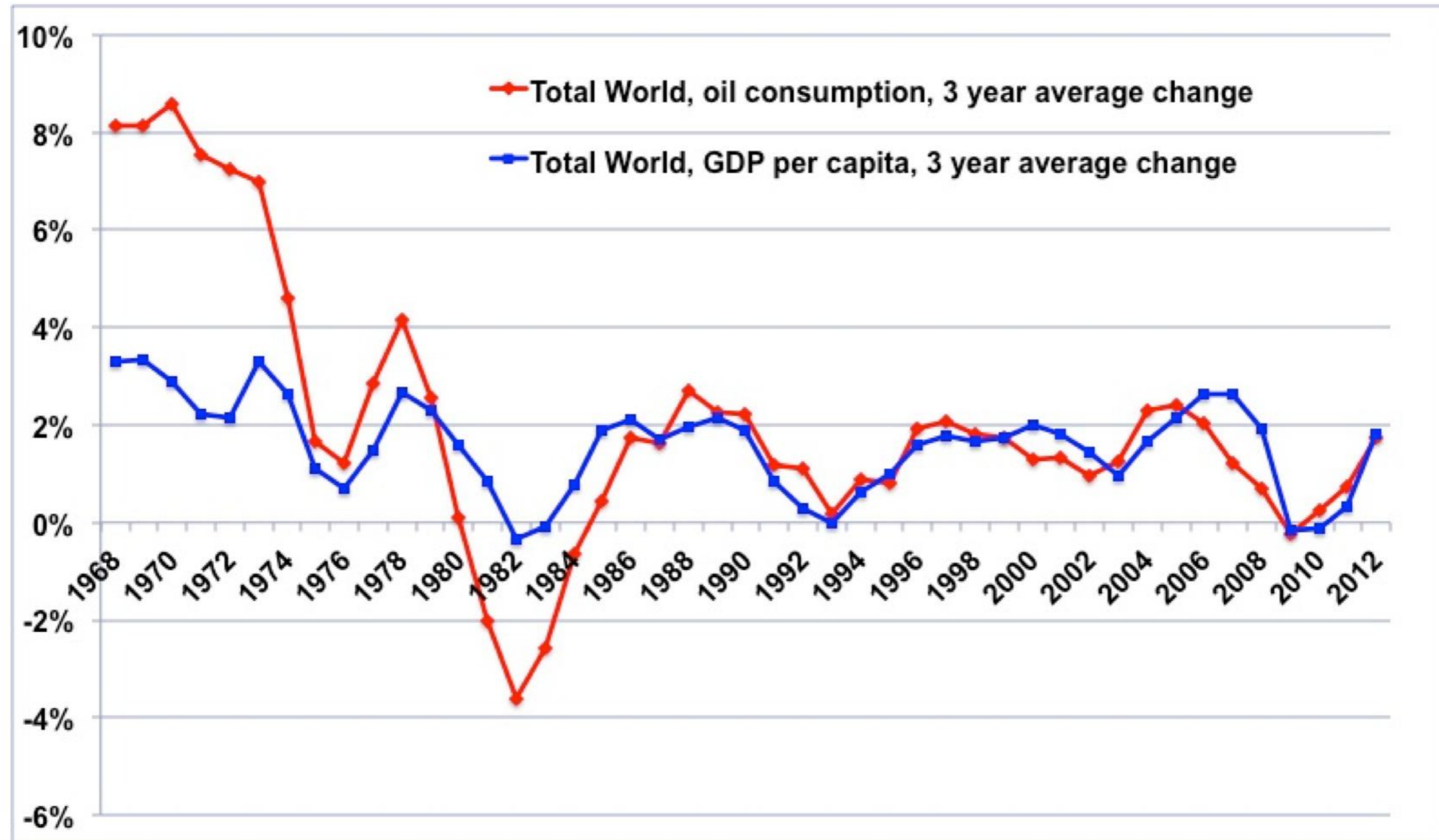


Energy & GDP



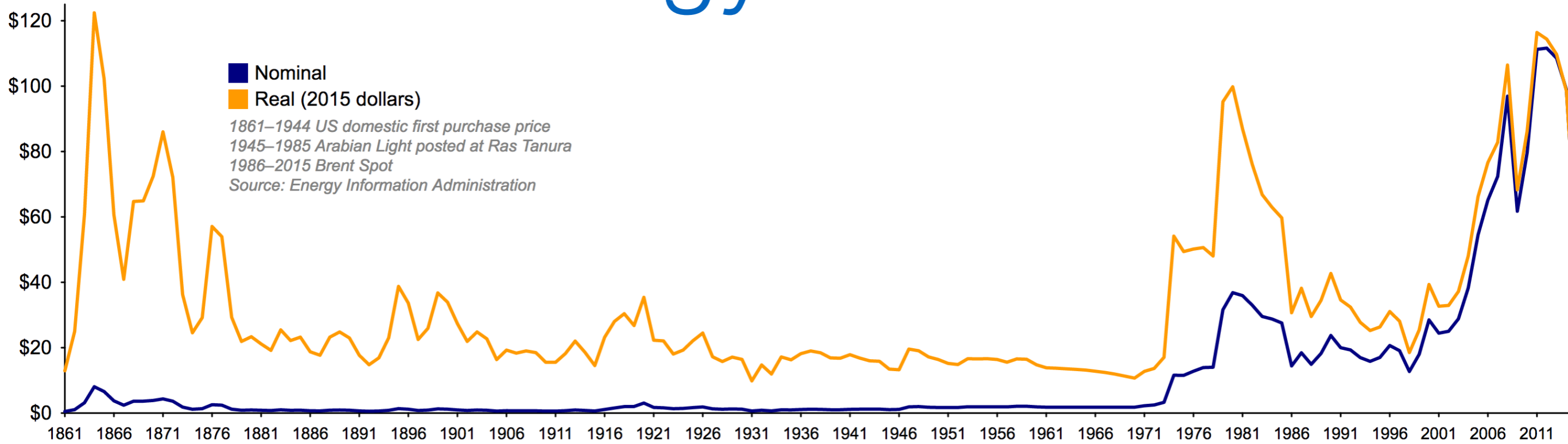
Source: The Shift Project - JM Jancovici

Energy & GDP

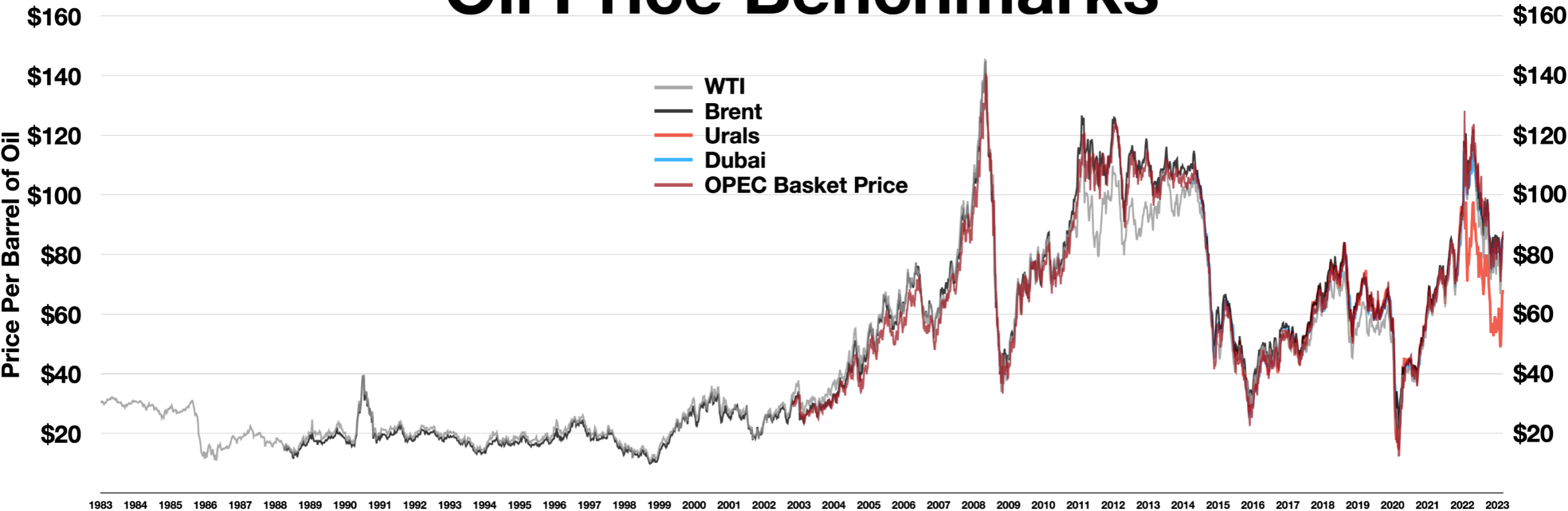


Variation lissée de la consommation mondiale de pétrole (rouge) et du PIB par personne (bleu). Source World Bank 2013 pour le PIB, BP Stat 2013 pour le pétrole

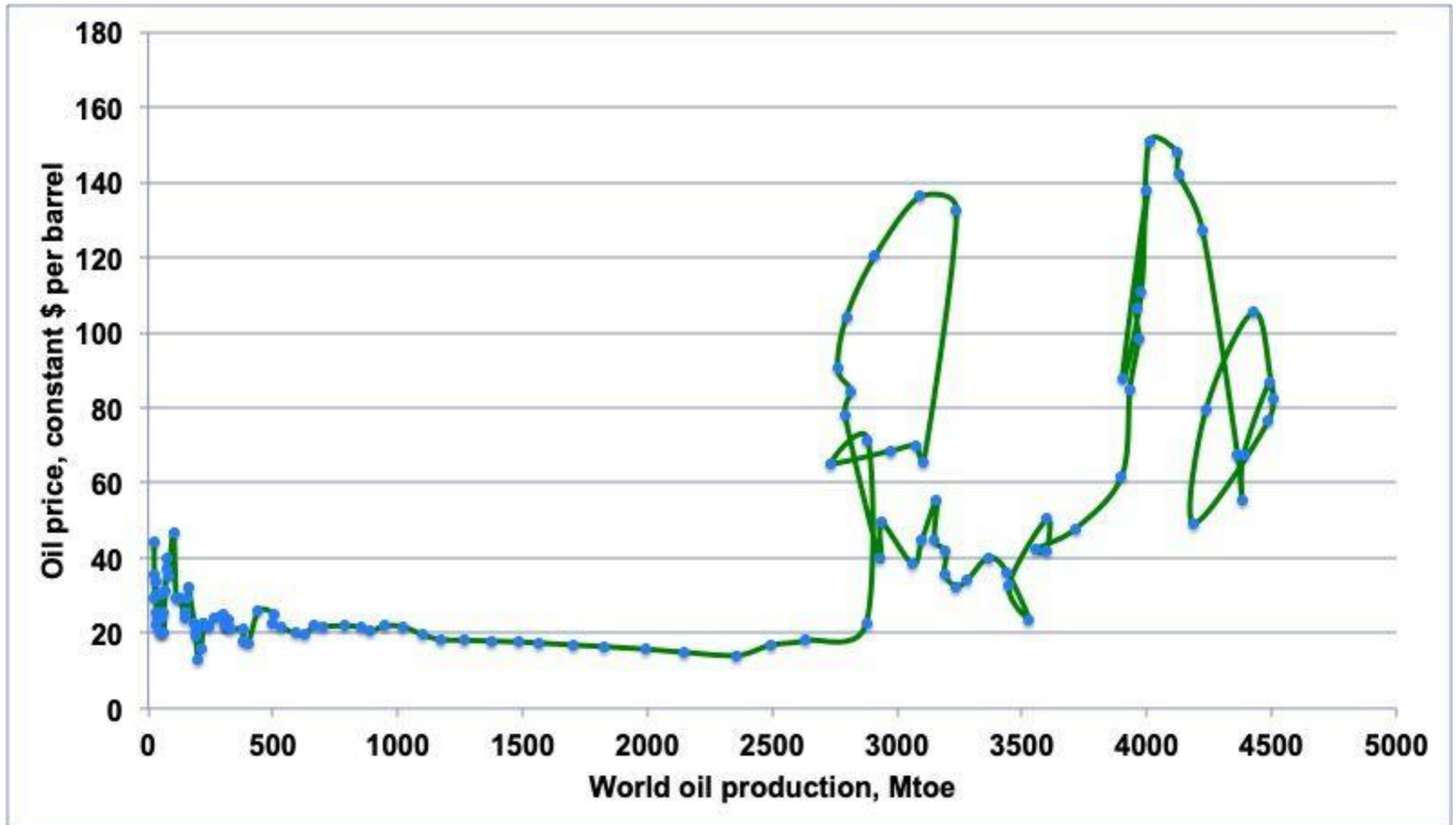
Energy & GDP



Oil Price Benchmarks



Demand & offer

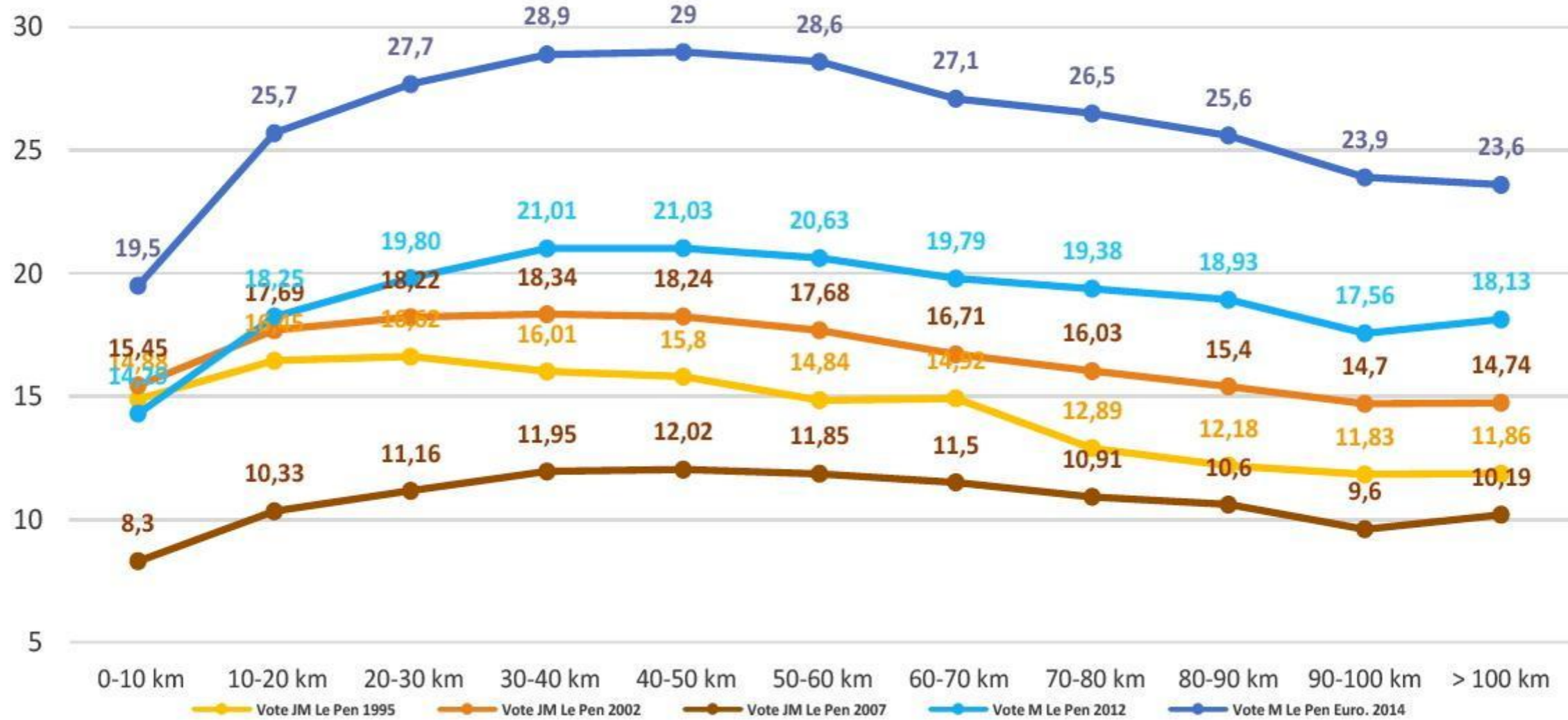


Each point represents a year between 1900 and 2023, and the points are "arranged" along the curve in chronological order, with 1900 on the far left. There is no simple link between the total quantity of oil sold and the price per barrel.

Source: [JM Jancovici, 2025](#)

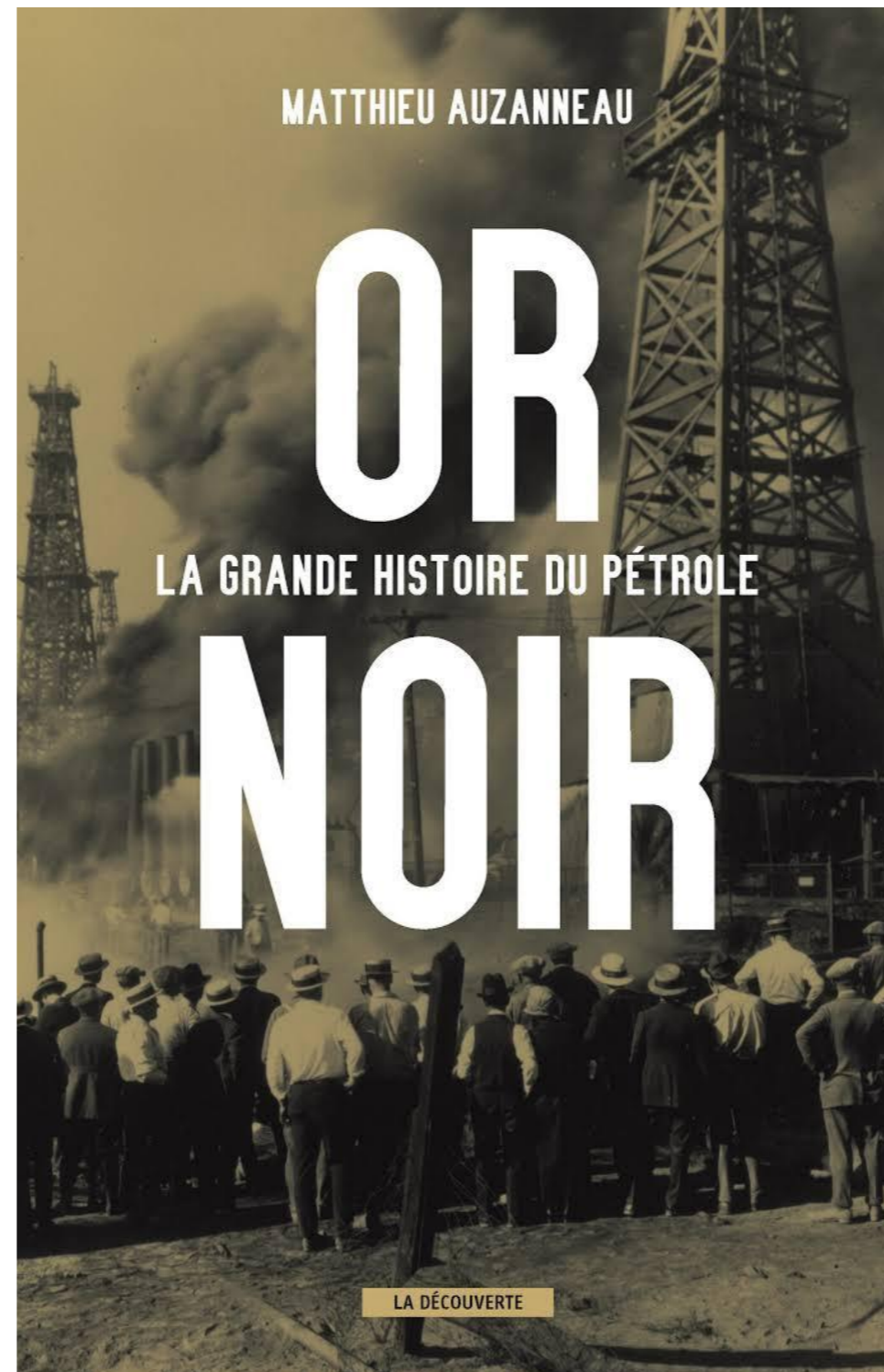
Energy & democracy

Score moyen du Front National en fonction de la distance aux villes



Quel lien avec l'énergie ?

A must read



The Challenge (1)

Non renewable

> 80% - < 20%

Renewable

The Challenge (2)

Dematerialized economy does not exist on its own.

« You cannot compute food, and even if you could, you would need an industry to build computers ».

D.R.

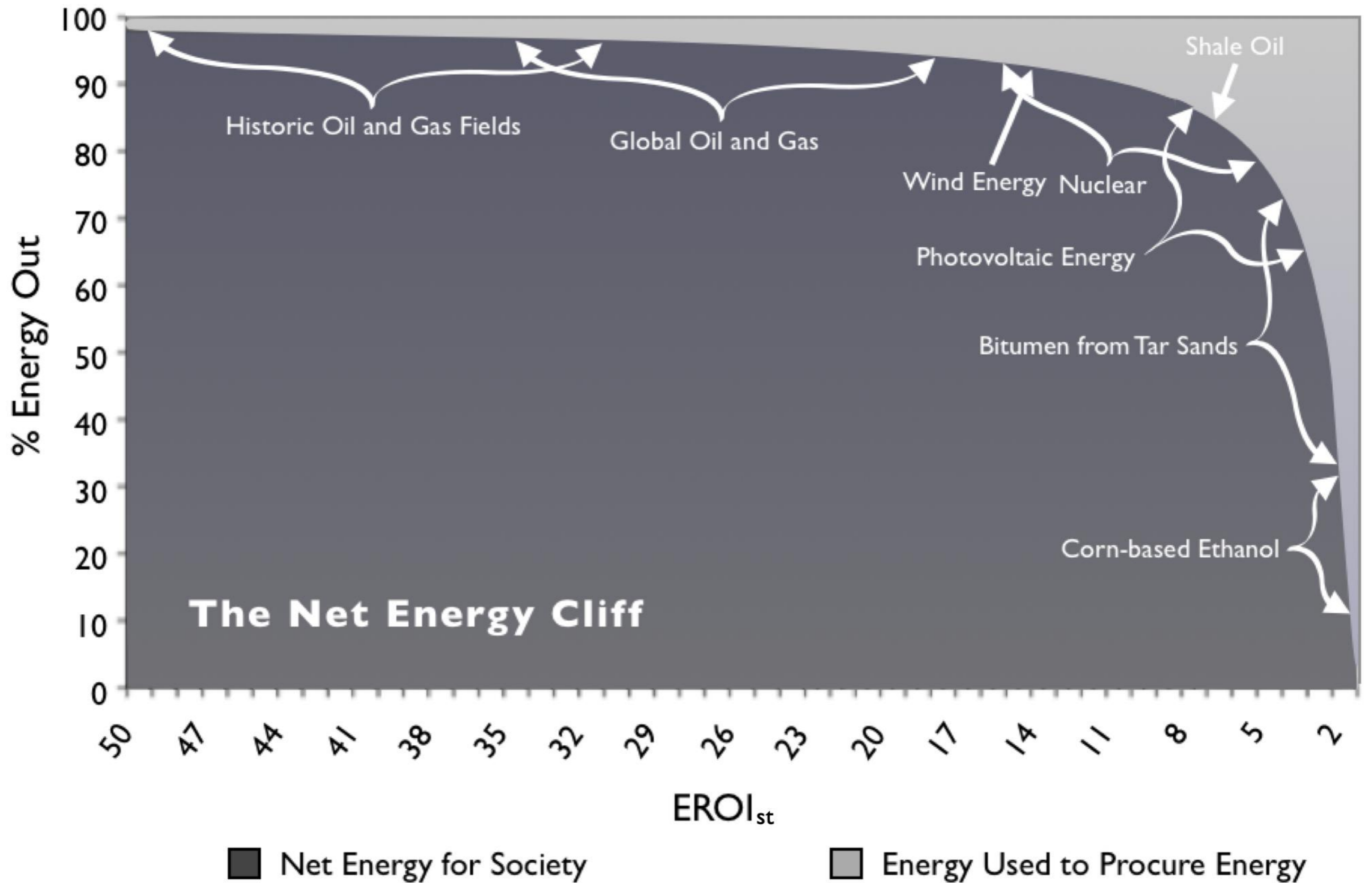
Modeling the transition

ERoEI

- **ERoEI for « Energy Return over Energy Investment »** (also called EROI) is the ratio of the amount of usable energy acquired from a particular energy resource to the amount of energy expended to obtain that energy resource:

$$EROI = \frac{\textit{Usable Acquired Energy}}{\textit{Energy Expended}}$$

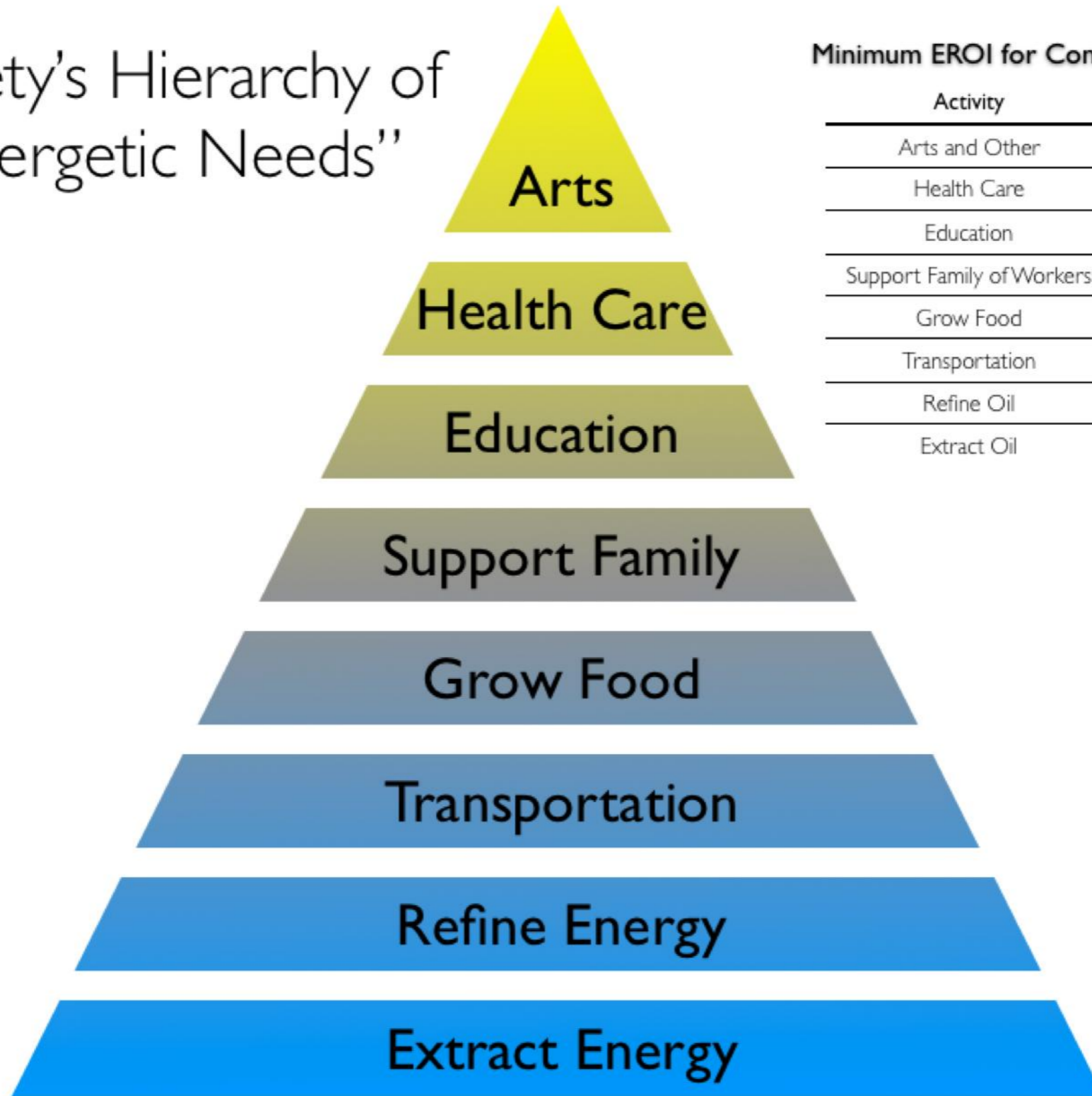
- The highest this ratio, the more energy a technology brings back to society
- Notation : 1:X



Source: *EROI of Global Energy Resources - Preliminary Status and Trends* - Jessica Lambert, Charles Hall, Steve Balogh, Alex Poisson, and Ajay Gupta State University of New York, College of Environmental Science and Forestry Report 1 - Revised Submitted - 2 November 2012 DFID – 59717

Note : some EROEI values have increased since 2012 (release of this graph), in particular solar PV.

Society's Hierarchy of "Energetic Needs"



Minimum EROI for Conventional Sweet Crude Oil

Activity	Minimum EROI Required
Arts and Other	14 : 1
Health Care	12 : 1
Education	9 or 10 : 1
Support Family of Workers	7 or 8 : 1
Grow Food	5 : 1
Transportation	3 : 1
Refine Oil	1.2 : 1
Extract Oil	1.1 : 1

Modeling the transition

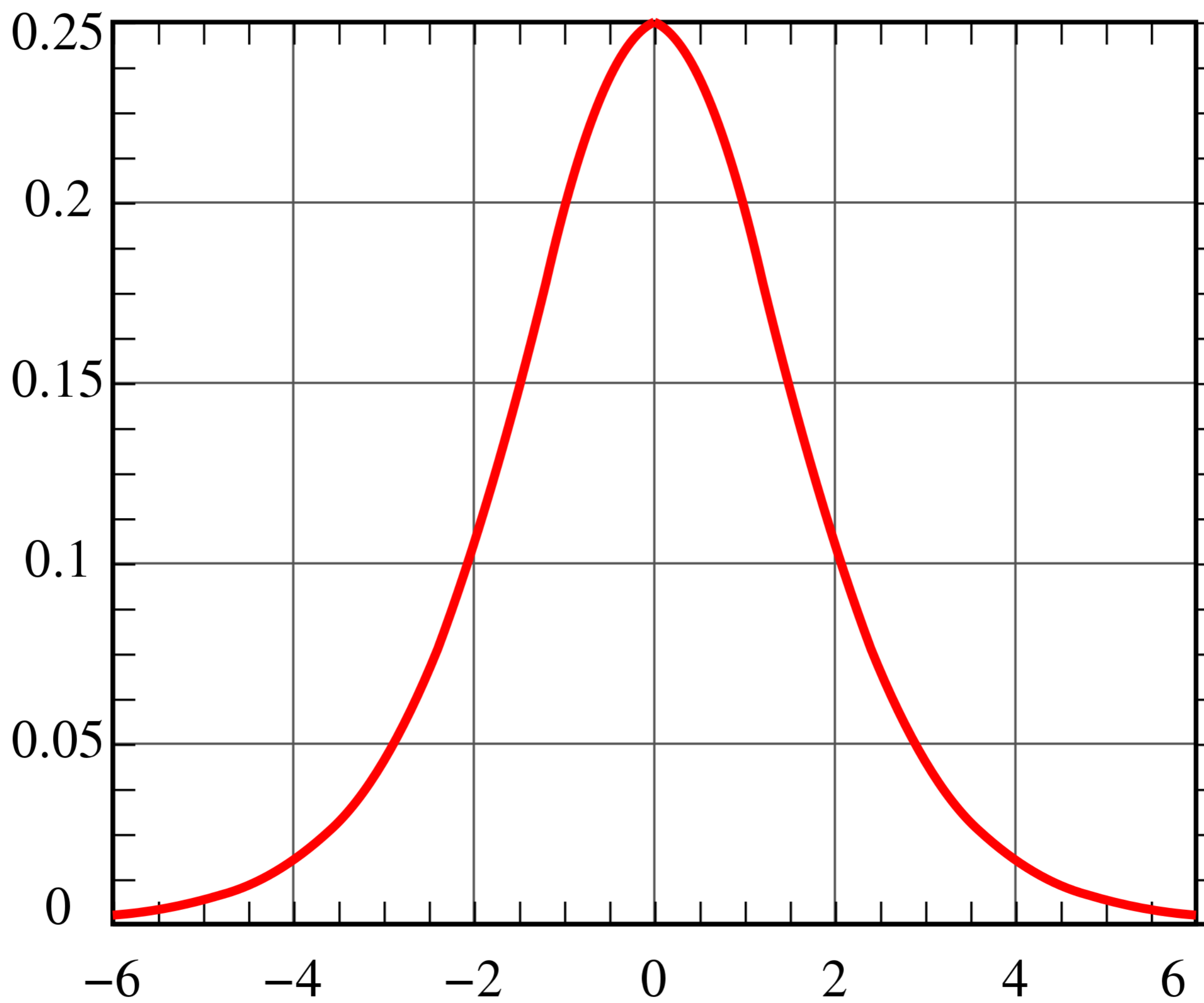
A discrete-time model of the deployment of « renewable energy » production capacities

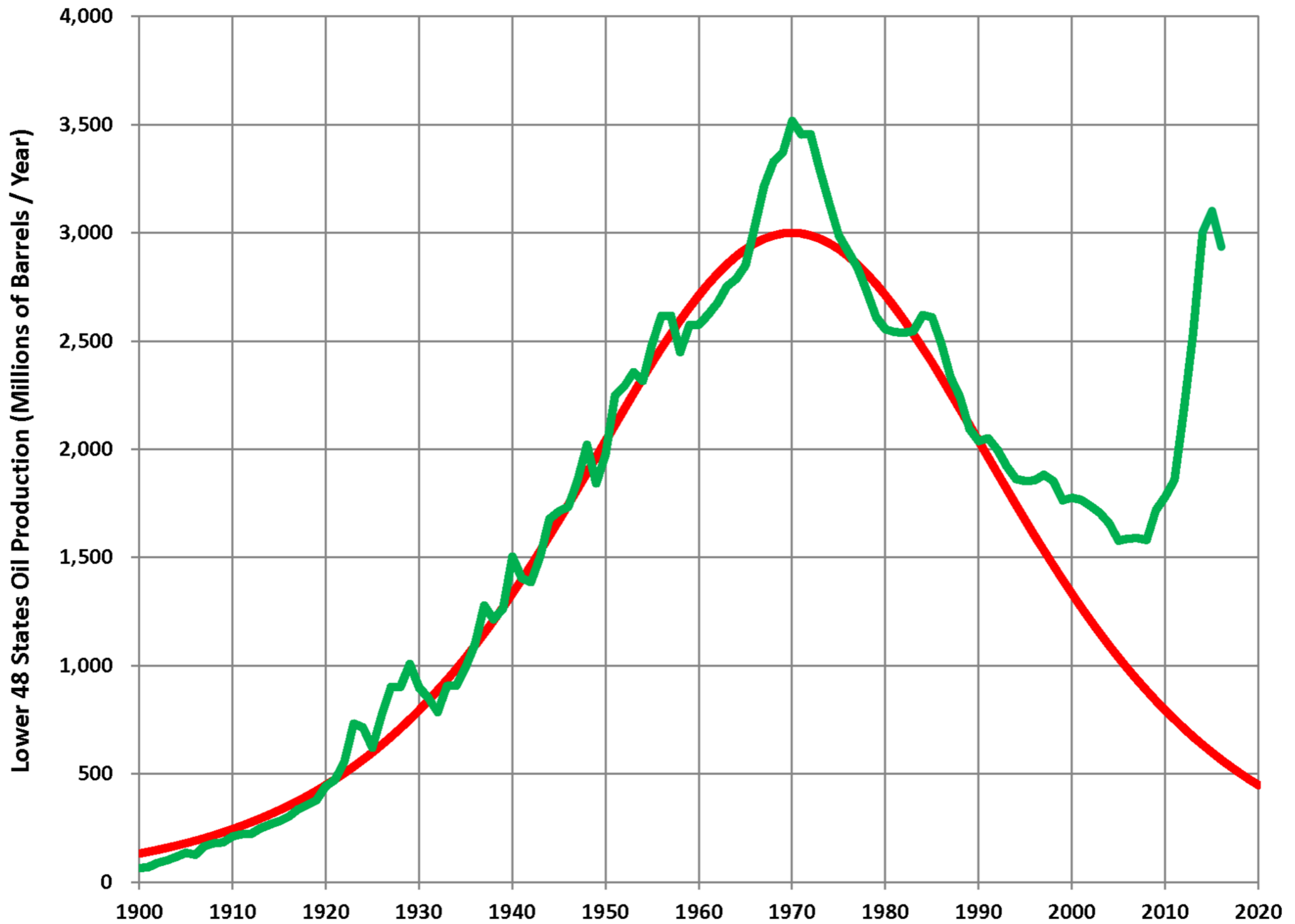
Budget of non-renewable energy

$$\forall t \in \{0, \dots, T - 1\}, B_t \geq 0$$

$$\exists r > 0, \exists \tau > 0, \exists t_0 \in \mathbb{R} : \forall t \in \{0, \dots, T - 1\},$$

$$B_t = \frac{1}{r} \frac{e^{\frac{-(t-t_0)}{\tau}}}{\left(1 + e^{\frac{-(t-t_0)}{\tau}}\right)^2}$$





Hubbert's curve is taken from M. King Hubbert (1956), using his upper-bound estimated ultimate recovery of 200 billion barrels. Actual lower-48 state production (total minus Alaska) is taken from the US Energy Information Administration website. Older production is from US Geological survey publications. Source: [Plazak via Wikipedia Commons](#).

Modeling the transition

Set of renewable energy production technologies:

$$\forall n \in \{1, \dots, N\}, \forall t \in \{0, \dots, T - 1\}, R_{n,t} \geq 0$$

Characteristics

$$\Delta_{n,t} \geq 0$$

$$ERoEI_{n,t} \geq 0$$

Deployment strategy

$$R_{n,t+1} = (1 + \alpha_{n,t})R_{n,t} \quad \alpha_{n,t} \in [-1, \infty[$$

Modeling the transition

Energy costs for growth and long-term replacement

$$\forall n \in \{1, \dots, N\}, \forall t \in \{0, \dots, T - 1\},$$

$$C_{n,t}(R_{n,t}, \alpha_{n,t}) \geq 0$$

$$M_{n,t} \geq 0$$

Total energy and net energy to society

$$\forall t \in \{0, \dots, T - 1\}, E_t = B_t + \sum_{n=1}^N R_{n,t}$$

$$S_t = E_t - \left(\sum_{n=1}^N C_{n,t}(R_{n,t}, \alpha_{n,t}) + M_{n,t} \right)$$

Modeling the transition

Constraint on the quantity of energy invested for energy production

$$\forall t \in \{0, \dots, T - 1\},$$

$$\exists \sigma_t : C_{n,t}(R_{n,t}, \alpha_{n,t}) + M_{n,t} \leq \frac{1}{\sigma_t} E_t$$

Modeling the transition

Further assumptions

- Energy cost for growth is proportional to growth, and done initially:

$$C_{n,t}(R_{n,t}, \alpha_{n,t}) = \frac{\Delta_{n,t}}{ERoEI_{n,t}} \alpha_{n,t} R_{n,t} \text{ if } \alpha_{n,t} \geq 0$$

- Long-term replacement cost is (i) proportional and (ii) annualized

$$M_{n,t}(R_{n,t}) = \frac{1}{ERoEI_{n,t}} R_{n,t}$$

$$E_0 = 1$$

$$B_0 = 0.85E_0$$

$$R_{1,0} = 0.01E_0$$

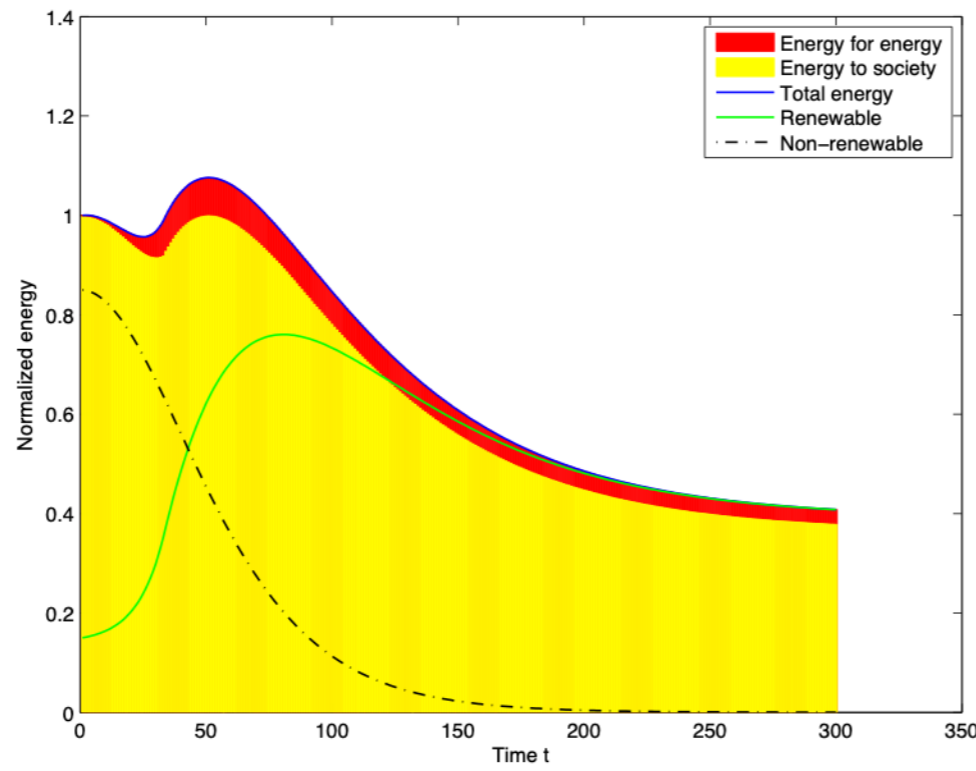


Fig. 2. Scenario “peak at time t=0”

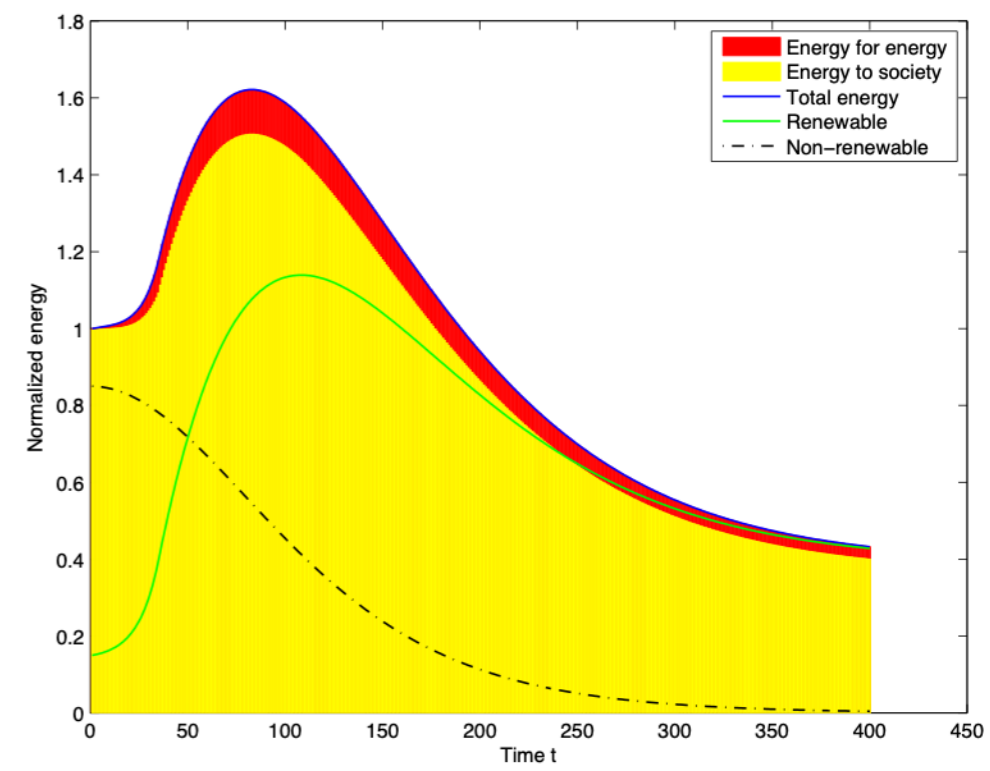


Fig. 3. Scenario “plateau at time t=0”

$$\sum_{n=2}^N R_{n,0} = 0.14E_0$$

$$ERoEI_{1,t} = 9$$

$$\Delta_{1,t} = 20$$

$$\sigma_t = 14$$

Constant growth
if possible, else
max admissible

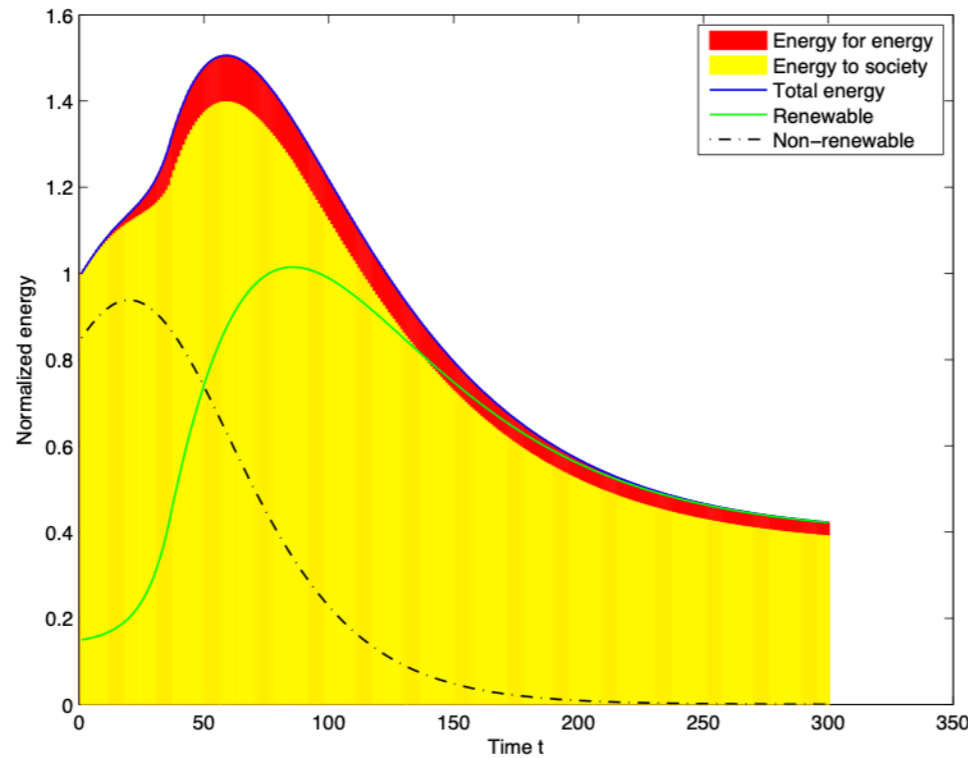


Fig. 4. Scenario “peak at time t=20”

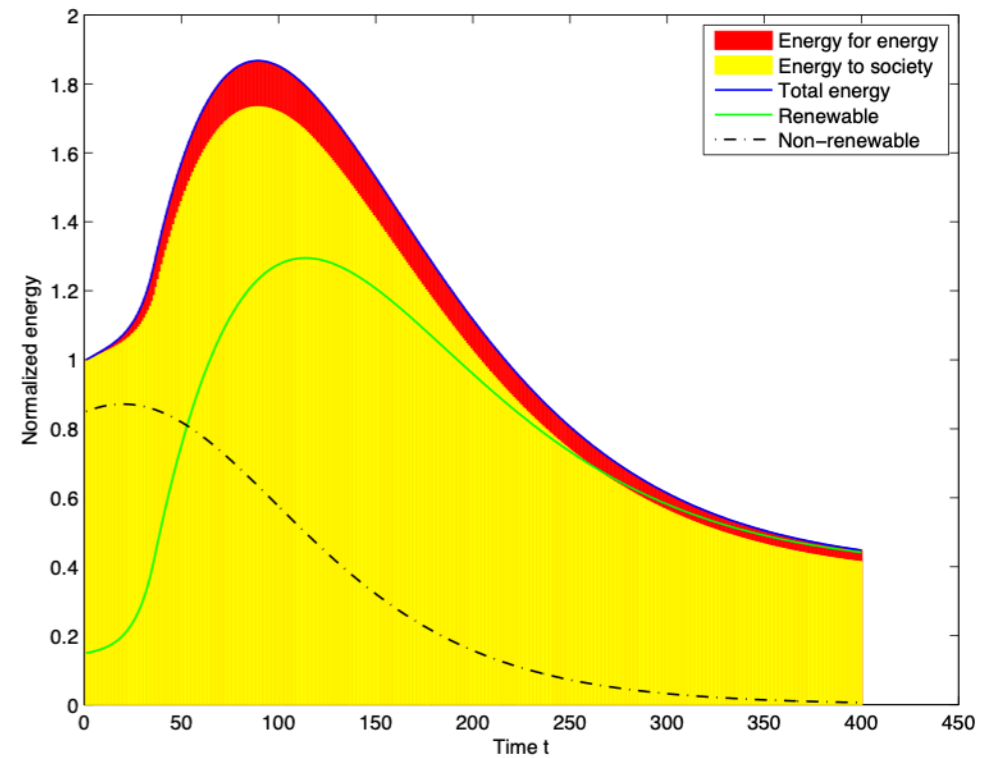
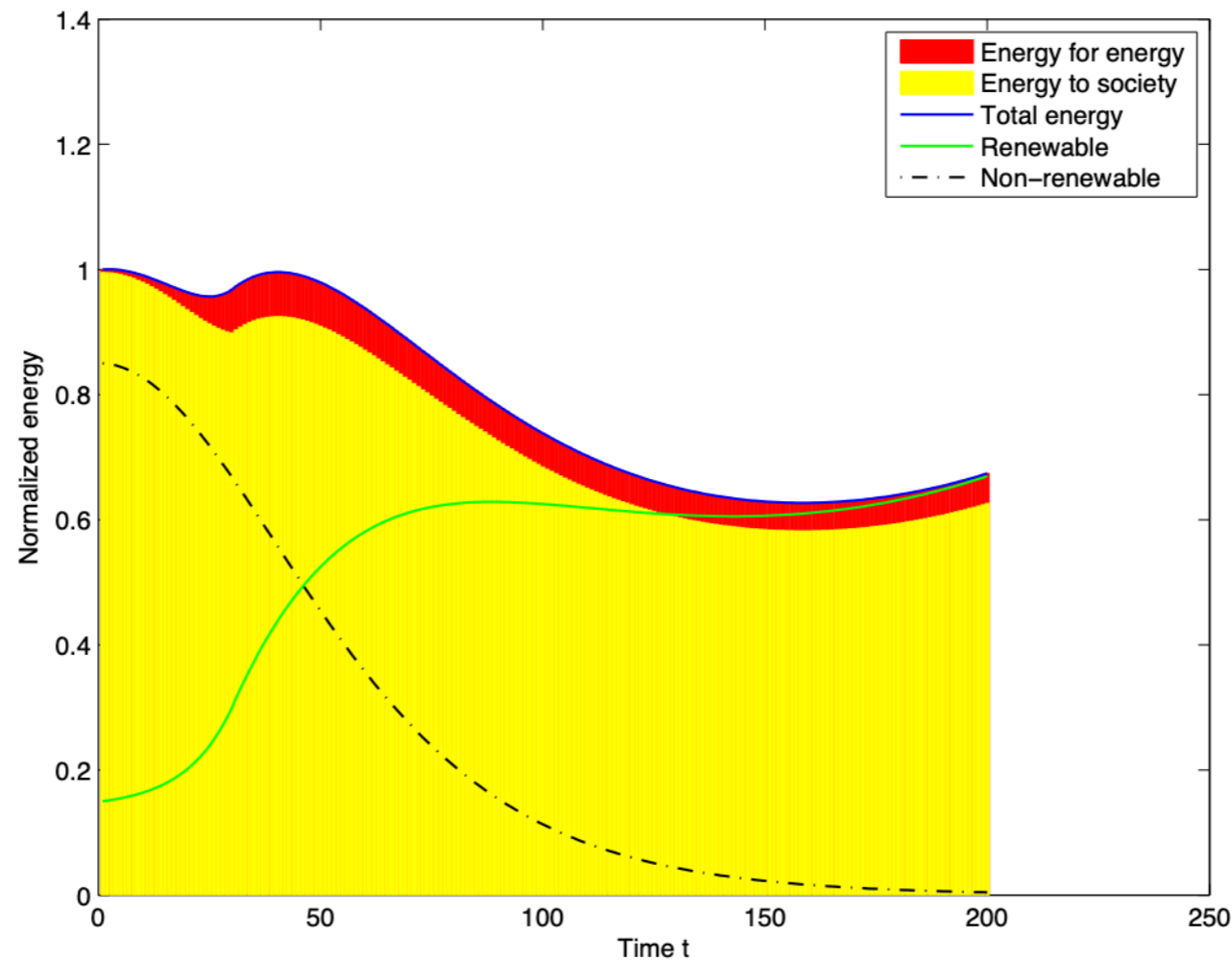


Fig. 5. Scenario “plateau at time t=20”

Modeling the transition

- Increasing the EROEI parameter



$$\forall t \in \{0, \dots, T - 1\}, EROEI_{1,t} = 9 + \frac{t}{T}(12 - 9)$$

So?

A few suggestions

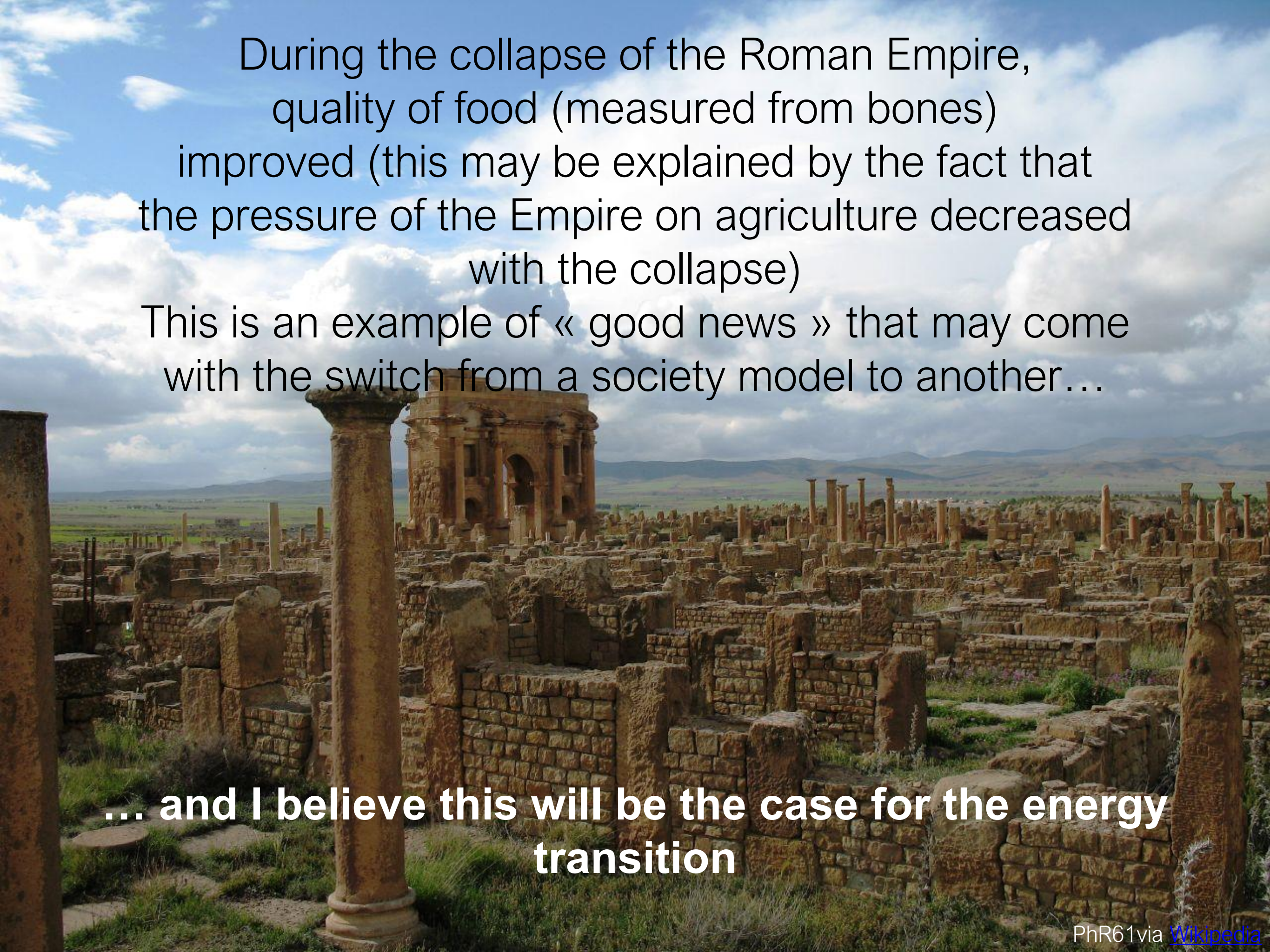
What kind of decisions can be suggested by such a « rough model »?

- Price may not always be a good indicator
- Pay attention to the EROEI
- Energy efficiency: « do better with less »
- But EROEI is not sufficient either...

The energy transition is a global process: how to prioritize actions?

- Back to Maslow

Epilogue



During the collapse of the Roman Empire,
quality of food (measured from bones)
improved (this may be explained by the fact that
the pressure of the Empire on agriculture decreased
with the collapse)
This is an example of « good news » that may come
with the switch from a society model to another...

**... and I believe this will be the case for the energy
transition**

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